

LOS ANGELES POLICE  
DEPARTMENT

INTERMEDIATE COLLISION  
INVESTIGATION

January 2005

INTERMEDIATE COLLISION INVESTIGATION SCHEDULE

TIME	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
0700 0750	COURSE INTRO & OVERVIEW	MATH REVIEW COMPLETED QUIZ #1	DRAG FACTOR & GRADE	TIME / DISTANCE Topic 6.0	
0800 0850	TIREMARKS & PHYSICAL EVIDENCE	EVIDENCE DOCUMENTATION	Topic 5.0	FIELD EXERCISE	GROUP PROJECT
0900 0950	Topic 2.0		SPEED COMPUTATION	SKID TESTING & DRAG SLED TESTS	CASE STUDIES
1000 1050	MATH REVIEW Topic 3.0	Topic 4.0	Topic 6.0		
1100 1130	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
1130 1530	MATH REVIEW CONTINUED Topic 3.0	SKID DOCUMENTATION FIELD PROBLEM	SPEED COMPUTATION CONTINUED Topic 6.0	SKID DOCUMENTATION & SPEED COMPUTATION FIELD PROBLEM	CASE STUDIES CONTINUED COURSE REVIEW FINAL EXAM
NOTES	SKILL ASSESSMENT	QUIZ #1 -10 PTS FIELD PROBLEM - 20 PTS		FIELD PROBLEM - 20 PTS	FINAL EXAM - 50 PTS

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EQUATION SHEET

TO FIND	WHEN GIVEN	EQUATION TO USE
<p style="text-align: center;">ACCELERATION (a) (feet per second<sup>2</sup>)</p>	<p style="text-align: center;">t      v<sub>i</sub>      v<sub>e</sub> t      v<sub>i</sub>      d v<sub>i</sub>      v<sub>e</sub>      d</p>	<p style="text-align: center;">1.      <math>a = \frac{v_e - v_i}{t}</math> 2.      <math>a = \frac{2d - 2v_i t}{t^2}</math> 3.      <math>a = \frac{v_e^2 - v_i^2}{2d}</math></p>
<p style="text-align: center;">INITIAL VELOCITY (v<sub>i</sub>) (feet per second)</p>	<p style="text-align: center;">t      a      v<sub>e</sub> t      a      d a      v<sub>e</sub>      d</p>	<p style="text-align: center;">4.      <math>v_i = v_e - at</math> 5.      <math>v_i = \frac{d}{t} - \frac{at}{2}</math> 6.      <math>v_i = \sqrt{v_e^2 - 2ad}</math></p>
<p style="text-align: center;">END VELOCITY (v<sub>e</sub>) (feet per second)</p>	<p style="text-align: center;">t      a      v<sub>i</sub> <i>a</i>      v<sub>i</sub>      d</p>	<p style="text-align: center;">7.      <math>v_e = v_i + at</math> 8.      <math>v_e = \sqrt{v_i^2 + 2ad}</math></p>
<p style="text-align: center;">DISTANCE (d) (feet)</p>	<p style="text-align: center;">t      a      v<sub>i</sub> a      v<sub>i</sub>      v<sub>e</sub> t      v<sub>i</sub>      v<sub>e</sub></p>	<p style="text-align: center;">9.      <math>d = v_i t + \frac{1}{2} at^2</math> 10.      <math>d = \frac{v_e^2 - v_i^2}{2a}</math> 11.      <math>d = \frac{t(v_i + v_e)}{2}</math></p>
<p style="text-align: center;">TIME (t) (seconds)</p>	<p style="text-align: center;">a      v<sub>i</sub>      v<sub>e</sub></p>	<p style="text-align: center;">12.      <math>t = \frac{v_e - v_i}{a}</math></p>

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**COURSE INTRODUCTION AND OVERVIEW**

**DEFINITIONS**

**TOPIC 1.0**

## LOS ANGELES POLICE DEPARTMENT INTERMEDIATE COLLISION INVESTIGATION

The intermediate Collision Investigation Course was designed to provide police officers additional, specialized knowledge of identification and documentation of physical evidence and speed computation procedures.

The goals of this course are to provide you, the student:

- With the knowledge and skills necessary to effectively conduct collision investigations through proper physical evidence identification and documentation.
- With the knowledge and skills necessary to use that physical evidence to estimate a minimum speed for the vehicles involved in the collision.

During this course you will:

- Understand the definitions and terminology used in the field of traffic collision investigation and reconstruction.
- Accurately identify and document different types of physical evidence.
- Identify the basis for each numeric component of speed estimate equations.
- Estimate the minimum speeds of vehicles based on physical evidence commonly encountered at traffic collisions.
- Complete time and distance calculations to determine position and sight line for drivers based on your speed estimates.

An outline of the course is presented on the next page. The course will use a mix of classroom instruction, group learning activities, and field projects. Your participation is paramount to the success of the course and will allow you to get the most benefit.

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COURSE OUTLINE

TOPIC NO.	CLASS	OBJECTIVE
1.0	INTRODUCTION & TERMINOLOGY	Present definitions of terminology used in traffic collision investigation and reconstruction
2.0	TIRE MARKS	Identification of different tire mark types, identification of other evidence. Photography review
3.0	MATH REVIEW	Present basic algebraic processes
4.0	PHYSICAL EVIDENCE DOCUMENTATION	Review physical evidence location and documentation procedures
5.0	DRAG FACTOR	Define the term "Drag Factor" and the different factors that effect the coefficient of friction of the roadway
6.0	SPEED COMPUTATION	Present the different methods of speed calculation based on physical evidence with the use of accepted equations
7.0	CASE STUDIES	Provide practical application in the use of the equations taught in Topic 6.0

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**I. DEFINITIONS AND TERMINOLOGY**

**A. ACCELERATION (a)**

The rate of change of velocity with respect to time. Acceleration may be either a positive value (acceleration), or a negative value (deceleration).

**B. CHORD (C)**

A straight line connecting two points on an arc.

**C. COEFFICIENT OF FRICTION OR MU (f or  $\mu$ )**

A number, expressed as a decimal, representing the resistance of an object sliding on a level surface.

**D. DISTANCE (d)**

A measurement between two points which can be expressed in any increment of measure (inches, meters, etc.). Generally will be expressed in decimal feet for this class.

**E. ENDING VELOCITY ( $v_e$ )**

An ending rate of motion during any change of position with respect to time. Can be expressed in any value of motion (miles per hour, meters per second, etc.). Expressed in feet per second for this class.

**F. ENERGY (E)**

The capacity to do work. There are various types of energy:

- 1) Potential Energy
- 2) Kinetic Energy
- 3) Heat Energy
- 4) Light Energy
- 5) Chemical Energy

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**G. FORCE (F)**

That which influences motion. A push or pull expressed in pounds.

**H. GRADE (G)**

The change in elevation in a specific direction along the centerline of a roadway or the path of a vehicle. Grade is positive (+) if the surface rises in the specified direction and negative (-) if it falls in that direction.

**I. GRAVITY (g)**

The force that pulls all things to the ground. The acceleration force of gravity is 32.2 feet per second per second (32 fps<sup>2</sup>).

**J. HEIGHT (h)**

A vertical distance measured in inches or feet. The distance from the apex of something upright to the surface the object is resting on.

**K. INITIAL VELOCITY (v<sub>i</sub>)**

A starting rate of motion during any change of position with respect to time. Can be expressed in any value of motion (miles per hour, meters per second, etc.). Expressed in feet per second for this class.

**L. KINETIC ENERGY (KE)**

Energy possesses by a body based on that object's motion.

**M. MASS (m)**

The amount of matter in an object. Measured by the object's weight divide by the acceleration of gravity.

**N. RADIUS (R)**

A line segment from the center of a circle and any point on its circumference.



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**O. RESULTANT COEFFICIENT OF FRICTION or DRAG FACTOR ( $f_r$ )**

An adjustment to the coefficient of friction or  $\mu$ , taking into account the roadway grade, superelevation, and  $f$  or the braking efficiency of the vehicle.

A number expressed as a decimal representing the coefficient of friction or  $\mu$ , plus or minus the grade of the surface times the braking efficiency of the vehicle.

**P. SPEED (S)**

The rate of progress, usually without regard to direction. Can be expressed in any value of motion (miles per hour, meters per second, etc.). Expressed in miles per hour for this class.

**Q. TIME (t)**

A measurement, generally in seconds with respect to collisions. The time over which a collision sequence occurs.

**R. VELOCITY (v)**

The rate of change of distance with respect to time. Can be expressed in any value of motion (miles per hour, meters per second, etc.). Expressed in feet per second for this class.

**S. WEIGHT (w)**

The product of mass times gravity. Can be expressed in any value of weight (pounds, kilograms, etc.). Expressed in pounds for this class.

**II. KEYS**

- A. In order to properly investigate and report traffic collisions, the investigator must have a clear understanding of the terminology that is accepted around the world in the traffic collision field.
- B. Knowledge of the terminology ensures accurate, consistent documentation and reporting.
- C. Knowledge of the basis for the variables used in the speed calculation equations enables an officer to use the equations in criminal cases and, if required, testify to them with confidence in court.

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**TIRE MARKS AND PHYSICAL EVIDENCE**

**TOPIC 2.0**

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**CHARACTERISTICS OF TIRE MARKS**

	SKID MARKS		ABS*	YAW	ACCELERATION	FLAT TIRE
	LOCKED	IMPENDING				
HEEL OTION	Slide, no roll	Retarded, rolling	Retarded rolling	Roll and slip	Slip and spin	Roll, no slip
PERATION	Braking	Braking	Braking	Steering	Accelerating	Rolling
NUMBER OF MARKS	Up to 4	Mostly 2, from rear tires	Up to 4	Up to 4	Usually 1, sometimes 2	Only 1, rarely 2
RIGHT AND LEFT TIRES	Equally strong	Equally strong	Equally strong	Front outside usually stronger	Equal if 2	Usually equal
CONTRAST	Front stronger	Negligible	Front slightly stronger	Rear fainter	Only driven wheels rotating	Equally clear
WIDTH	If straight, same as tire	If straight, same as tire	Same as tire	Varies, 1" - 1'	Same as tire	Tire tread edge marks
BEGINNING	Usually faint	Faint	Faint	Strong	Strong	Varies
END	Usually abrupt	Start of locked tire mark	Faint	Strong	Very gradual	Varies
VARIATIONS	Always parallel to the mark	Always parallel to the mark	* Always parallel to the mark	Always oblique	Always parallel to the mark	None
OTHER DETAILS	Outer edges often stronger on front		Dissipate quickly	Side rib marks may show	Outer edges often stronger	May include rim marks if tire unseated from rim

\* When there is steering input by the driver, the tire marks left by a vehicle equipped with ABS are unique insofar as they may be different types of marks. When a vehicle is equipped with ABS, each wheel has a individual sensor This may cause one tire to leave a locked wheel mark while another leaves a yaw mark.

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Tire marks are the most important type of physical evidence left by the vehicles at a traffic collision scene because they tell the complete story of how the collision occurred, and they do not lie.

The collision investigator must properly identify and interpret the tire marks in order to accurately report the scene physical evidence for later analysis by either them (after this class), or a trained collision reconstructionist.

An important part of physical evidence documentation is photography. Different photography techniques must be used in different situations in order for the photographs to properly depict the scene as it was at the time of the investigation.

I. Basic Principles of Physics – In the 1680's, Sir Isaac Newton studied and defined how objects move and interact with one another:

### A. Newton's First Law of Motion:

"A body will remain at rest or in uniform motion unless it is acted upon by an unbalanced force"

1. When a vehicle skids, it loses directional stability and slides in a straight line until it comes to a stop. This is an example of directional uniform motion. The vehicle comes to a stop because of the friction between the tires and the roadway (the unbalanced force).
2. Now let's add a collision into the mix. Without a collision, the vehicle described above will slide in a straight line. If another vehicle collides with it while it is skidding (an unbalanced force), its direction will change.
3. These changes in direction in tire marks are very important for collision investigators in that they can be used to establish the Area of Impact

### B. Newton's Second Law of Motion

"The acceleration of an object is directly proportional to the applied force and inversely proportional to its mass" or "If the net force acting on a body is not zero, the body will be accelerated in the direction of the force"

This is why in traffic collisions, "size and speed wins".

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C. Newton's Third law of Motion

"For every action there is an equal and opposite reaction." or "Whenever one body exerts a force upon a second body, the second body exerts force upon the first."

This concept is used in more advanced accident reconstruction to determine the speeds of the vehicles and is beyond the scope of this class.

II. Tire Mark Definition and Characteristics

A. Tire mark definition:

1. A mark left on any surface by a tire through friction.
2. The term "tire mark" includes all evidence of marking.

B. The three basic methods of leaving visible tire marks:

1. Extreme Deceleration

- a. Braking causing the wheels to cease rotation.
- b. Damage causing the wheels to cease rotation.

2. Extreme Change of Direction

- a. From an intentional effort on the part of the driver
- b. From an impact/contact with another object/vehicle

3. Extreme Acceleration

- a. Propelling force or thrust generated exceeds the pavements frictional resistance.
- b. "Peeling Out": Accelerating at a rate where the rear tires lose traction with the roadway.

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C. What tire marks indicate

1. The vehicle's actions
  - a. Braking
  - b. Accelerating
  - c. Turning
2. In some cases, the minimum speed of the vehicle.
3. The vehicle's direction of travel.
4. Location of the vehicle on the roadway surface.
5. Point or area of impact.
6. Possible mechanical failure.

III. Types of Tire Marks

A. Impending: Wheel rotating slower than the forward motion of the vehicle

B. Locked Wheel

1. Non-rotating wheel, moving in a straight or curved line in the original direction travel.
2. If tires in line with the skid, striations will be parallel to the direction of travel.
3. Front vs. Rear characteristics
  - a. Front are darker on the outside edges and striations are lighter in color due Overdeflection.
  - b. Rear have light outside edges and darker striations due to Underdeflection.
  - c. Overdeflection and underdeflection caused by weight shift around the vehicle's pitch axis during heavy braking.

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4. Four Wheel Locked Overlapping vs. Four Wheel Locked Independent
  - a. Four wheel locked overlapping tire marks occur when the rear tires skid and mark directly over the marks left by the front tires. When calculating speed from four wheel locked, overlapping tire marks, deduct the wheelbase or a standard 10 feet from the total skid length.
  - b. Four wheel locked independent tire marks occur when all four skids can be seen independent of one another. No deduction of 10 feet for the wheelbase should be taken when applying the tire marks to a skid chart.
  
5. Scrub Marks
  - a. A locked, braked, or rotating wheel of a vehicle sliding in other than a forward direction.
  - b. Usually left after impact, except when known to be caused by centrifugal force.
  
6. Yaw Marks
  - a. Rotating tire mark made from a vehicle turning at a rate greater than the frictional force of the roadway.
  - b. Striations are perpendicular to the direction of travel.
  - c. The rear tire must track to the outside of the front on the same side of the vehicle.
  
7. ABS Tire Marks
  - a. A mark left by a vehicle equipped with an Anti-lock Braking System.
  - b. The marks are generally very light in consistency and hard to see.
  - c. The tire motion is kept in the impending mode due to the application-release action of the brakes.
  - d. If turning during an ABS application, the resulting tire marks may look like the conventional yaw mark on the outside of the turn, and a conventional locked wheel tire mark on the inside of the turn.

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8. Skip Skids

- a. A mark left by a locked tire that loads and unloads on the roadway surface.
- b. This can be due to weight shift, a poor suspension or an uneven roadway.
- c. The gaps in the marks must be less than 5 feet.

9. Gap Skids

- a. A mark left by locked tires where the brakes have been applied, released and then reapplied.
- b. The gaps in the marks must be over 5 feet.

10. Acceleration Marks

- a. A mark left on the roadway surface when the propelling force or thrust generated by the vehicle exceeds the roadway surface's frictional resistance causing the tires to spin.
- b. Can resemble a locked wheel tire mark in that the striations will be parallel to the direction of travel. Differences are:
  1. Weaved in nature.
  2. Start dark and end light and uneven.
  3. May see shards of rubber and roadway removed at the start of the mark.

IV. Other types of marks on the roadway

A. Gouge marks

1. Left by contact between a hard portion of the vehicle and the roadway (undercarriage, wheel, etc.), where roadway material is removed.
2. Occurs at Maximum Engagement where the vehicles can no longer crush, but have momentum that has to be dispelled. Generally the vehicles will be forced downward onto the roadway, causing the gouge marks.
3. Good indicator of the Area of Impact.



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**B. Scrape Marks**

1. Left by contact between a hard portion of the vehicle and the roadway (undercarriage, sheet metal during rollover, etc.), where no roadway material is removed.
2. Can indicate post-impact direction and orientation of the vehicles.

**C. Fabric transfer**

1. Left when an ejected motorist, pedestrian, bicyclist or motorcyclist slides across the roadway.
2. Friction between the clothing and the roadway will tear fabric from the clothing or leave a colored "haze".

**D. Human tissue transfer**

1. Also left when an ejected motorist, pedestrian, bicyclist or motorcyclist slides across the roadway.
2. Friction will tear exposed skin from person leaving a transparent deposit of tissue.
3. May or may not have blood with it.

**E. Vehicle fluids**

1. Spatter, pooling and runoff will indicate post-impact direction of travel, orientation and at rest position.
2. Document path as you would a tire mark.

**V. Photographing Tire Marks**

**A. Procedure of photographing**

1. During daylight hours:
  - a. Take overall photographs of the tire marks in 20-foot increments, unmarked with chalk, as they were when you arrived at the scene. These photographs should follow the path of the tire marks.
  - b. Mark the starting points of the tire marks with chalk.

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- c. Mark the tire marks at 10-foot intervals from the point of impact back to the start of the tire mark.
  - d. Mark the ending points of the tire marks.
  - e. Mark the post impact tire marks from the point of impact forward to the end of the tire marks.
  - f. Re-photograph the marks from start to end at 20-foot intervals.
2. During darkness hours:
- a. Identify types of lighting available at the scene
    - 1) 35 MM flash unit
      - a) Short range
      - b) 10 - 12 feet
    - 2) Separate flash unit
      - a) Short to long range
      - b) As far as 150 feet, depending on the unit used
    - 3) Flash light or vehicle head lights
      - a) Short to medium range
      - b) Range depends on the amount of light supplied
    - 4) BE AWARE OF THE LIMITS OF YOUR FLASH UNIT
  - b. Nighttime photographing procedure:
    - 1) Take overall photographs of the tire marks in 10-foot increments, unmarked with chalk, as they were when you arrived at the scene. These photographs should follow the path of the tire marks.
    - 2) Mark the starting points of the tire marks.
    - 3) Mark the tire marks at 10-foot intervals from the point of impact back to the start of the tire mark.

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- 4) Mark the end points of the tire marks.
  - 5) Mark the post impact tire marks from the point of impact forward to the end of the tire marks.
  - 6) Re-Photograph the marks from start to end at 10-foot intervals. Follow through the point of impact to the ending point of the marks.
  - 7) Remember, you want a logical depiction of the tire marks from start to finish in both a unmarked state and then marked for identification
3. Other areas to photograph
- a. The at rest positions of the vehicles: These photos should be taken depicting the vehicles in relation to each other or a recognizable landmark.
  - b. Vehicle damage and identification.
  - c. Damage to objects
  - d. Other evidence:
    - 1) Scrapes or gouges in the roadway.
    - 2) Fluid from vehicles or other sources
    - 3) Physical features of the roadway, including any vision obscurement, etc.

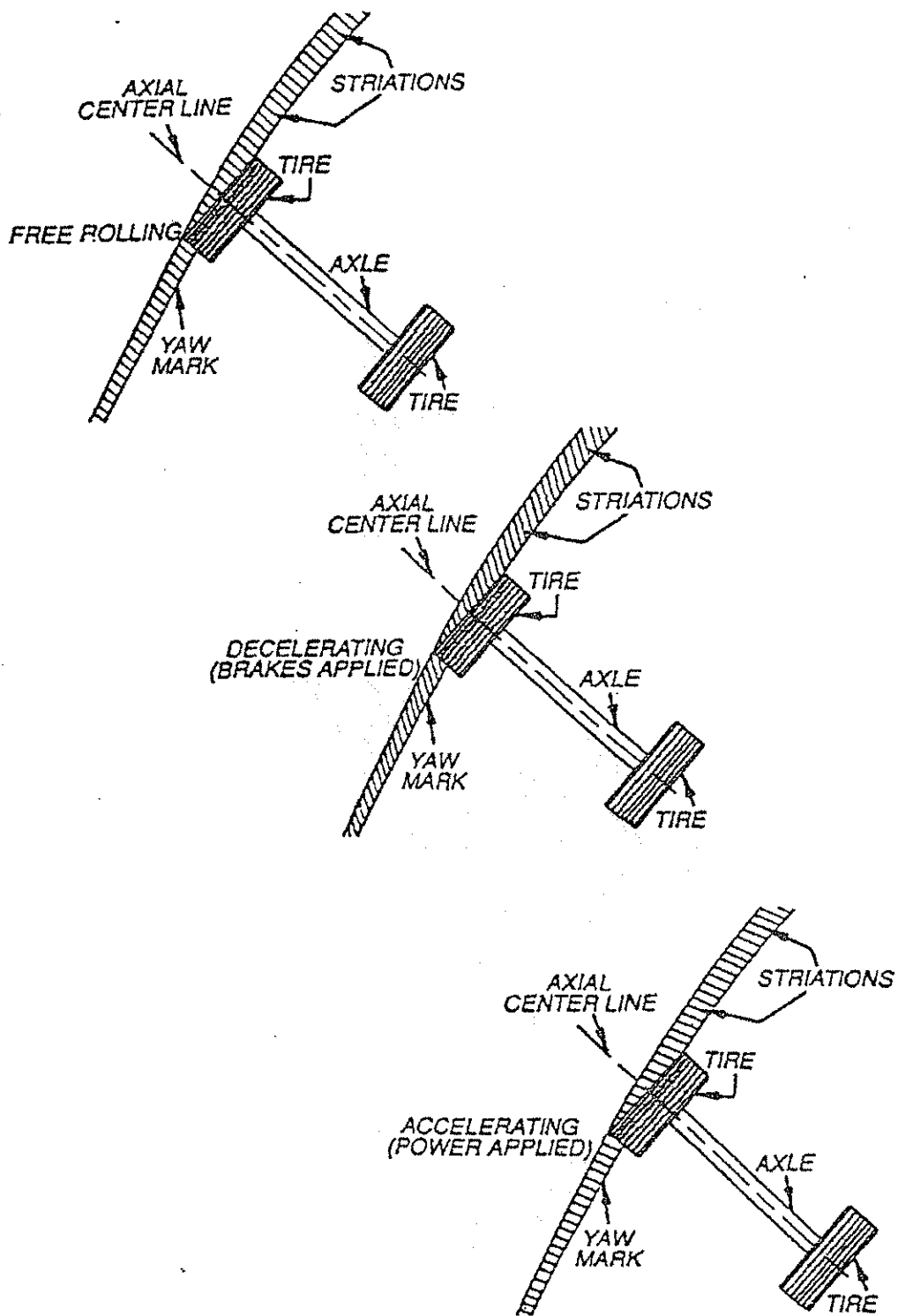
**VI. KEYS**

- A. Officers assigned to Collision Investigation units cannot be mere report takers. They must be investigators.
- B. Knowledge of the characteristics of the physical evidence one may encounter at the scene of a traffic collision is necessary:
  1. To ensure accurate reporting and documentation of the collision.
  2. Because the speed calculations that are going to be learned this week are based on proper interpretation of the physical evidence.

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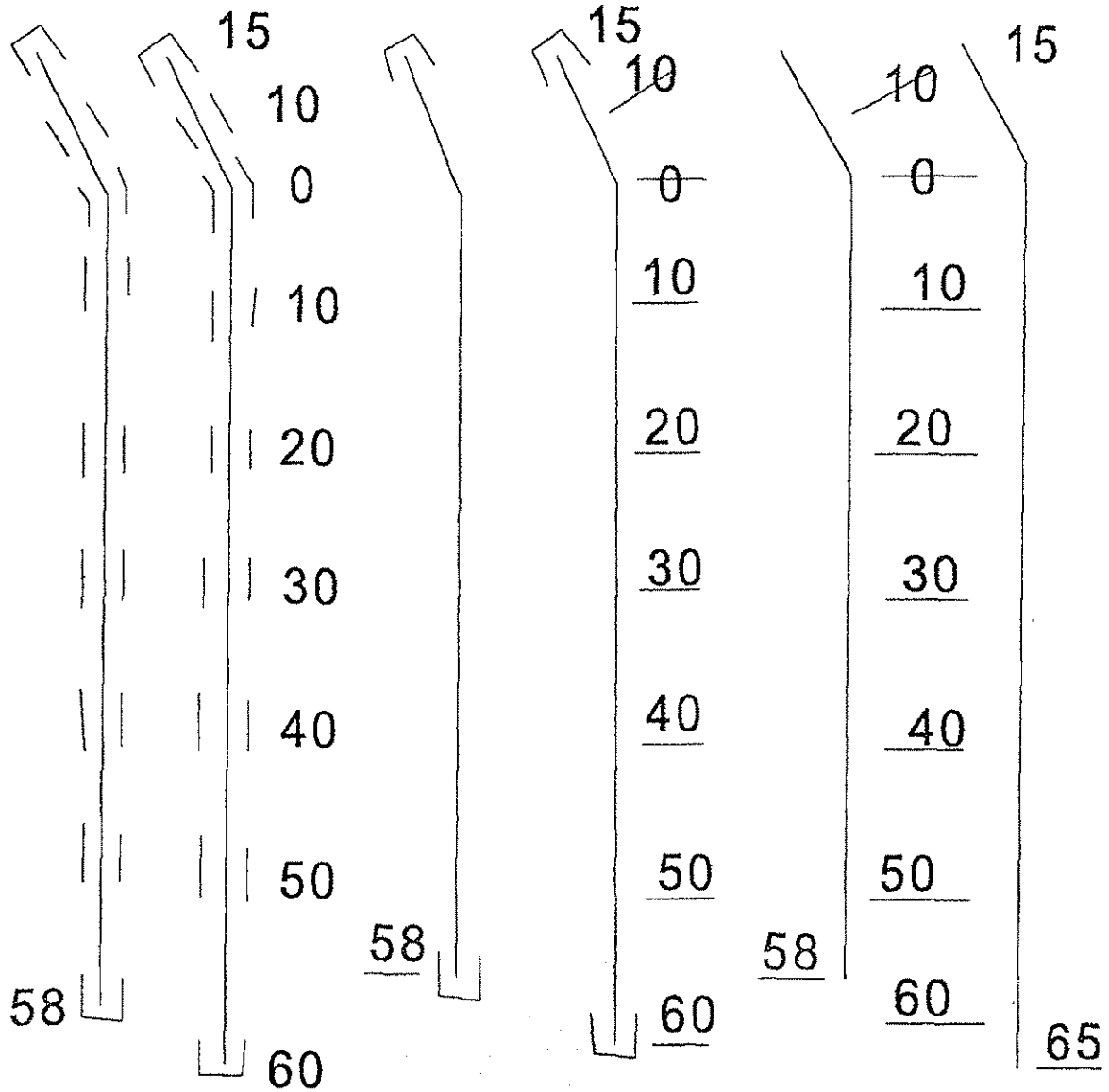
3. For court testimony. Once calculations have been completed and are used to establish a violation for which one is to be prosecuted, they may have to be explained to a jury. One of the main points of contention by the defense is usually the interpretation of the physical evidence (type of tire mark) and then its use to calculate the minimum speed of the vehicle.

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MARKING AND PHOTOGRAPHING  
TIRE MARKS



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**TIRE MARK GLOSSARY**

**ACCELERATION** – The propelling force or thrust generated in an amount exceeding the pavement's frictional resistance, made from the drive wheel or wheels of the vehicle.

**CENTRIFUGAL SCUFF** – See YAW MARK.

**CHIP** – A short, shallow removal of surface material typically made by a strong, sharp metal object under pressure, usually without striations.

**CHOP** – A broad, shallow removal of surface material typically made by a strong, broad, sharp metal object under pressure, usually with striations.

**CHORD** – A straight line connecting two points of an arc or circle.

**COEFFICIENT OF FRICTION or MU** – A number, expressed as a decimal, representing the resistance of an object sliding on a level surface, divide by the weight of the object.

**COLLISION SCRUB** – a short, usually broad tire mark made during and after engagement of the vehicles.

**CRITICAL SPEED SCUFF** – See YAW MARK.

**DRAG FACTOR** – A number, expressed as a decimal, representing the coefficient of friction, plus or minus the grade of the surface, times the braking efficiency of the vehicle. (Also referred to as the Resultant Coefficient of Friction).

**GAP SKID** – A braking skid mark that is interrupted by release and re-application of the brakes.

**GOUGE** – A short to long, shallow to deep, removal of surface material by a hard, sharp pointed object.

**GRADE** – The change in elevation in a specified direction along the center line of the roadway or the path of a vehicle, expressed in percent.

**IMPENDING TIRE MARK** – The mark left by a tire whose rotation is slower than the forward motion of the vehicle, usually preceding a locked skid mark.

**MIDDLE ORDINATE** – A line perpendicular to a chord, connecting the mid-point of the chord to a point on the arc.

**NOMOGRAPH** – A graph in which three or more scales are arranged where a straight line drawn through the values on any two scales will cross the third scale at a corresponding value; a graphic calculator.

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**OVERDEFLECTION** – A condition in which the pressure of a tire is greater at the outside edges than the middle of the tread; an overloaded, weight shift or underinflated condition.

**SCRAPE** – An Area of a hard surface covered with broad scratches made by a sliding metal part without great pressure. No surface material is removed.

**SCUFF MARK** – A friction mark made on a surface by a tire which is both rotating and slipping; acceleration scuffs, critical speed scuffs, flat tire marks.

**SCRUB MARK** – A locked, braked, or rotating tire of a vehicle sliding in other than a forward direction, except when known to be caused by centrifugal force.

**SIDESLIP** – See YAW MARK

**SKID MARK** – A frictional mark on a surface made by a tire that is sliding without rotation. The sliding of the tire may be due to braking, collisions damage or other circumstances.

**SPEED SKID CHART** – See NOMOGRAPH

**SKIP SKID** – A braking skid mark interrupted at frequent, regular intervals caused by the loading and unloading of weight on the tires.

**STRIATIONS** – Narrow parallel streaks or scratches, usually made by friction or abrasion on the roadway or other surface.

**SUPERELEVATION** – The vertical distance between the heights of the inside and outside edges of a roadway.

**TRACK WIDTH** – The distance between the center of the tire tread on one side of the vehicle and the center of the tire tread on the opposite side of the vehicle.

**WHEELBASE** – The distance from the center of the axle of the front wheels to the center of the axle of the rear wheels.

**YAW MARK** – A mark left on a surface by a rotating tire of a vehicle in a turning movement where the centrifugal force exceeds the frictional force of the surface.



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**MATH REVIEW**

**TOPIC 3.0**

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This class is designed to review the concepts of math and basic algebra. The material will include:

- Addition
- Subtraction
- Multiplication
- Division
- Order of operations
- Equation solving
- Powers and radicals'
- The use of calculators

At the end of the class, the successful student will understand and be able to apply the basic concepts of algebra to collision investigation equations with the use of a calculator.

I. Ground Rules

A. There may be several of you that have taken algebra courses in school. This class may be a review and seem simple to you, but may be somewhat harder for others. If you have an algebra background, we may ask you to assist with some of the students that are having problems.

B. Notes on notation

1. Multiplication signs: "x" or "•" or "()"

a.  $4 \times 3 = 12$

b.  $4 \bullet 3 = 12$

c.  $4(3) = 12$ ; This will be the notation used in this course

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2. Division sign: “÷” or “ $\frac{x}{y}$ ” or “ $\overline{)}$ ”

a.  $12 \div 4 = 3$

b. 
$$\begin{array}{r} 3 \\ 4 \overline{)12} \end{array}$$

c.  $\frac{12}{4} = 3$  ; This notation will be used in this course

3. Radical sign:  $\sqrt{\quad}$

a. Used to notate the operation of finding the square root of the number under the radical

b. This is the number, that when multiplied by itself, equals the number under the radical

Example:  $\sqrt{81} = 9$

4. Exponents:  $x^2$

a. Used to notate the operation of multiplying the base number by itself

b. The only exponent used in this course will be the base number squared

Example:  $9^2 = 81$

C. Use of a calculator in this class will definitely be permitted

1. Do not use the calculator to get ahead of the class

2. Avoid use of the calculator's "arithmetic logic"

a. Work out each step

b. This enables the instructors to find any problems in the mathematical order of operations you may have had

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II. There are four basic types of calculators:

A. Basic

1. Handles basic math operations as entered by the user.
2. Does not utilize the Fundamental Order of Operations when completing a series of calculations.

B. Arithmetic Logic

1. Handles basic math operations as entered by the user.
2. Utilizes the Fundamental Order of Operations without user instruction.
  - a. For algebraic equations to produce the correct answer, the user has to follow a strict order of simplification while working toward the final result.
  - b. Within each equation, the user must simplify in the following order:
    - 1) Parenthesis
    - 2) Exponents
    - 3) Multiplication
    - 4) Division
    - 5) Addition
    - 6) Subtraction

C. Scientific

Same as the Arithmetic Logic with several additional functions such as:

1. Exponentiation and square roots
  - a. A single key is set to calculate the exponent function.
  - b. A single key is set to calculate the square root function.

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2. Logarithms

- a. Exponent of the power to which it is necessary to raise a fixed number (the base) to produce the given number.
- b. For example, the logarithm of 100 (base 10) is 2 because  $10^2$  equals 100.

3. Trigonometric Functions

- a. An area of mathematics involving triangles.
- b. Trigonometric calculations use the relationships between the sides and the angles of triangles to calculate position, distance, speed, and many other things.
- c. A variation on the scientific calculator replaces the scientific functions with statistical functions such as Standard Deviation, 'Sigma' functions, etc.

4. Programmable

- a. Same as Scientific (or Statistical).
- b. Have a limited memory that allows formulae to be entered and recalled.

III. Determining Your Calculator's Functions

A. Enter the following equation exactly as given:

2 <plus button> 3 <multiplication button> 4 <equal button>

1. If the answer is 20, then you have a Basic Calculator only.
2. If the answer is 14, then you have a calculator with Arithmetic Logic.
3. If you have any other answer, you did something wrong. Go back to the start and try again.

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- B If your calculator has Arithmetic Logic, you will no longer have to concern yourself with the Fundamental Order of Operations in your calculations.

Example:

$$50 - 2 + 4 * 3 * 2 =$$

Solution: (as your calculator "sees" it)

$$50 - 2 + 24 =$$

$$48 + 24 =$$

Answer: 72

- C. You will still have to watch for parentheses.

#### IV. Basic Mathematical Principles

##### A. Exponents

1. A small figure placed to the upper-right of a number showing the number of times to multiply that number by itself.
2. This class will only use the square exponent.

Example:  $9^2 = 81$  is the same as writing  $9(9) = 81$ . We multiply the base number (9) by itself, one time.

3. Determining the square of a number (by use of the calculator)
  - a. Enter the number you are seeking to find the square of

- b. Push the Square Key 

$x^2$
-------

Example: Enter 9; Press 

$x^2$
-------

; The answer should show 81

- c. Some calculators may have to use the "Inverse" or "2<sup>nd</sup>" function to calculate the square.

Example: Enter 9; Press 

2 <sup>nd</sup>
-----------------

 or 

INV
-----

; then 

$\sqrt{\quad}$
----------------

;

The answer should show 81

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**B. Multiplication**

1. May be seen as described before, two numbers multiplied together notated by parenthesis

Example:  $4(3) = 12$

2. May also be seen in an equation as two variables (symbols) placed next to each other

Example:  $d = vt$       Where:  $d =$  distance in feet  
 $v =$  velocity in feet per second  
 $t =$  time in seconds

To calculate the distance, one has to multiply the velocity factor by the time factor

**C. Division**

Will be notated as one number "over" another. Always divide the top number, the numerator, by the bottom number, the denominator

Example:  $\frac{12}{3} = 4$       Divide the bottom number into the top number

**D. Addition - Adding the factors in an equation**


**E. Subtraction - Subtracting the factors in an equation**

**F. Square Root**

1. The root of a number is another number that, when multiplied by itself, equals the original number.
2. Determining the root of a number (by use of the calculator)
  - a. Enter the number you are seeking to find the root of

- b. Push the Square Root Key



Example: Enter 81; Press ; The answer should show 9

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- c. Some calculators may have to use the "Inverse" or "2<sup>nd</sup>" function to calculate the square root

Example: Enter 81; Press  $\boxed{2^{\text{nd}}}$  or  $\boxed{\text{INV}}$  ; then  $\boxed{x^2}$  ;

The answer should show 9

V. The rules of algebra

- A. The rules of algebra show us how to rearrange symbols in an equation. Typically, accident reconstruction formulas allow us to calculate a quantity, such as speed in the slide-to-stop equation:

$$S = \sqrt{30 df}$$

- B. In this equation, several symbols appear, with the unknown quantity represented by a symbol that is isolated all by itself on the left side of the equation. All of the known quantities in the equation are either numbers or symbols representing values that have been obtained through measurements, or that have been supplied to us. To solve for the unknown quantity, we simply substitute in the numbers and do the mathematical operations indicated by the formula. For instance, if  $f = 0.7$  and  $d = 84$  feet, we have:

$$S = \sqrt{30 df}$$

$$S = \sqrt{30(84)(0.7)}$$

$$S = \sqrt{1764}$$

$$S = 42 \text{ MPH}$$

- C. Unfortunately, when we begin with the underlying physical principles that lead to the reconstruction formulas, the unknown quantity usually isn't isolated on the left-hand side initially. We therefore must use the rules of algebra to move the various symbols around until the unknown symbol is all by itself on the left-hand side.



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D. Not many rules are required, but for those who have not used algebra extensively, some practice is required to become proficient at applying those rules. We will express each rule in words, then symbolically, and give a numerical example.

1. Rule 1. If two quantities are equal, one can be substituted for the other.

If  $a = b$ , and  $a = x$ , we can also write  $b = x$

Example: From the definitions class:

$$KE = \frac{1}{2}mv^2 \quad \text{and} \quad m = \frac{w}{g}$$

Therefore:

$$KE = \frac{1}{2} \left( \frac{w}{g} v^2 \right) \quad \text{or} \quad KE = \frac{wv^2}{2g}$$

2. Rule 2. If a quantity is subtracted from its equal, the result is zero

If  $a = b$ , then  $a - b = 0$

To isolate the unknown:

$$\begin{aligned} X + 4 &= 36 \\ X + 4 - 4 &= 36 - 4 \\ X &= 32 \end{aligned}$$

3. Rule 3. If a quantity is divided by its equal, the result is one

If  $a = b$ , then  $a/b = 1$

Example:

$$\begin{aligned} d &= vt \\ \frac{d}{t} &= \frac{vt}{t} \\ \frac{d}{t} &= v(1) \end{aligned}$$

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4. Rule 4. Adding (or subtracting, multiplying, dividing) the same amount to equal quantities results in two new quantities that are still equal

$$\text{If } a = b, \text{ then } a + c = b + c$$

Example: If two cars have the same weight (e.g.,  $a = 3200$  lbs and  $b = 3200$  lbs), then  $a = b$ , and adding the same weight to each (a driver whose weight is given by  $c = 160$  lbs) will produce occupied cars whose weights are the same:

$$a = b$$

$$a + c = b + c$$

$$3200 \text{ lb} = 3200 \text{ lb}$$

$$3200 \text{ lb} + 160 \text{ lb} = 3200 \text{ lb} + 160 \text{ lb}$$

$$3360 \text{ lb} = 3360 \text{ lb}$$

5. Rule 5. Adding (or subtracting) 0 to a quantity does not change its value

$$a + 0 = a \quad \text{and} \quad a - 0 = a$$

6. Rule 6. Multiplying (or dividing) a quantity by 1 does not change its value

$$(a)(1) = a \quad \text{and} \quad a/1 = a$$

7. Rule 7. The order of multiplying (or adding) two numbers is immaterial

$$ab = ba \quad \text{and} \quad a + b = b + a$$

Example:  $(3)(7) = 21$  and  $(7)(3) = 21$ . Likewise,  $3 + 7 = 10$  and  $7 + 3 = 10$

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8. **Rule 8. The order of equality is immaterial**

$$\text{If } a = b \text{ then } b = a$$

Example: If, in an original equation, the unknown quantity is on the right hand side, this rule allows us to place it on the more familiar left hand side

If  $ab = x$  and we know that  $a = 3$  and  $b = 5$ , then  $(3)(5) = x$ , or  $x = (3)(5) = 15$

9. **Rule 9. The product of the sum is the sum of the products** (Slide 30)

$$a(b + c) = ab + ac$$

Example: If  $a = 3$ ,  $b = 4$ , and  $c = 5$ , we can write:

$$\begin{aligned} a(b+c) &= (3)(4+5) \\ &= (3)(9) \\ &= 27 \end{aligned}$$

But we also have:

$$\begin{aligned} ab+ac &= (3)(4)+(3)(5) \\ &= 12+15 \\ &= 27 \end{aligned}$$

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10. Rule 10. The quotient of a sum is the sum of its quotients

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Example: If  $a = 4$ ,  $b = 8$ , and  $c = 2$ , we have:

$$\frac{a+b}{c} = \frac{4+8}{2} = \frac{12}{2} = 6$$
$$\frac{a}{c} + \frac{b}{c} = \frac{4}{2} + \frac{8}{2} = 2+4 = 6$$

Note: It is not true that:

$$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$$

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- E. These various rules can be applied to an equation where the unknown quantity is not all by itself. For example, if, in the equation, a known quantity is added to the unknown quantity, we can subtract the known quantity from both sides

Example: The equation from basic physics that relates the acceleration of an object to it's change in velocity is given by:

$$a = \frac{v_e - v_i}{t}$$

Apply the rules of algebra to this equation to find the final velocity if we know the acceleration, time and initial velocity

Solution: We want to manipulate the equation so that the unknown quantity,  $v_e$ , is all by itself on one side of the equation. To do that, we can first get rid of the  $t$  in the denominator by multiplying both sides of the equation by the same quantity,  $t$ , and using the fact that equals divided by equals produces 1:

$$\begin{aligned} a &= \frac{v_e - v_i}{t} \\ at &= \frac{v_e - v_i}{t} t \\ at &= \frac{v_e - v_i}{t} t = (v_e - v_i)(1) \\ at &= v_e - v_i \end{aligned}$$

We now need to get rid of the  $v_i$  that is with the  $v_e$ . We can do that by adding  $V_i$  to both sides of the equation:

$$\begin{aligned} at &= v_e - v_i \\ at + v_i &= v_e - v_i + v_i \\ at + v_i &= v_e + 0 \\ at + v_i &= v_e \end{aligned}$$

We can now swap the two sides of the equation to get the unknown,  $v_e$  all by itself:

$$\begin{aligned} v_e &= at + v_i \\ v_e &= v_i + at \end{aligned}$$

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VI. Equation Solving – One Unknown

A. Addition and subtraction are inverse operations

EXAMPLES:

1)  $x + 3 = 8$

2)  $5 + y = 13$

3)  $6 + a = 18$

4)  $x - 10 = 2$

5)  $y - 3 = 14$

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B. Multiplication and division are inverse operations

EXAMPLES:

1)  $3x = 12$

2)  $4y = 16$

3)  $16b = 32$

4)  $6x = 30$

5)  $\frac{x}{3} = 12$

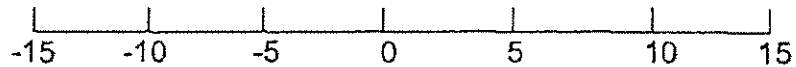
6)  $\frac{y}{4} = 10$

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VII. Negative Numbers

In the equations we will be using there will be negative numbers such as negative acceleration (deceleration).

A. Number Line



B. Multiplication of negative numbers

1. Multiplication of positive numbers will result in a positive answer.

Example:  $5(3) = 15$

2. Multiplication of a positive number and a negative number will result in a negative answer.

Example:  $5(-3) = -15$

3. Multiplication of a negative number and a positive number will result in a negative answer.

Example:  $-6(6) = -36$

4. Multiplication of a negative number and a negative number will result in a positive answer.

Example:  $-5(-5) = 25$

5.  $(5 + 3)(2) + 6(5 - 8)(3) =$

6.  $3(x - 26) - 30 =$

Where  $x = 12$